

Change in OptiCAL Colorant Tags for ICC Compatibility

Michael H. Brill, Dana Gregory, and Brian Levey (22 September 2003)

Since its introduction more than 6 years ago, OptiCAL has transformed the chromaticities of the R, G, and B primaries (called colorant tags) in a way that is not “ICC-standard”, but does have the advantage of transforming the entire source gamut (RGB triangle) to the entire destination gamut. That method was selected because it represented the best compromise between recommended best practice and compliance to the rather ambiguous ICC specification available at the time. While OptiCAL became the preferred monitor calibration software for professionals and consultants in the photography, animation, designer, and prepress industries, this discrepancy went unobserved until Spring 2003, when it became widely discussed in several professional forums.

In response, ColorVision has implemented, as an OptiCAL default, the Bradford transform for chromatic adaptation of the colorant tags, as recommended by the ICC. On the advice of the color consultants that supported us in this project, we have maintained the legacy “non-ICC compliant” method as an option that can be selected from an “ICC Settings” dialog in OptiCAL.

The change to the new Bradford algorithm makes very little difference in the colors you see on a monitor, and will in any case affect only the most saturated colors, not the skin tones and gray scales to which viewers are most sensitive. But with the change, the colors will now be correctly handed off (through the colorant tags) to a destination device that “speaks ICC.”

Compliance tests at ColorVision compared the new and legacy versions of OptiCAL, and brought into the comparison two other popular monitor-calibration software packages offered by other suppliers. The legacy OptiCAL and the two other software packages all failed the compliance test, but the new version of OptiCAL passed. Among screen-calibration software suites, OptiCAL is one of the first to have implemented the correct colorant-tag adaptation relative to the Photoshop application (which is ICC compliant).

The preview, now available on our web site, has been beta tested by a community of color consultants who were extremely helpful and supportive. In particular, we thank Bruce Fraser, Ethan Hansen, Bruce Lindbloom, Ian Lyons, Chris Murphy, Parker Plaisted, Andrew Rodney, and David Tobie.

I. ICC Compatibility

By using the new version of OptiCAL with your monitor, you can be confident of retrieving the same color on an identical monitor at the other end of a communication chain, so long as all the software in the chain is “ICC compatible.”

What is ICC compatibility? The International Color Consortium (ICC) agreed on a format for color information transfer between display devices, so the information is correctly interpreted and applied regardless of its source (see www.color.org). The ICC's method was to require the source to deliver its color specifications in the pre-agreed "Profile Connection Space" (PCS). The PCS is a sort of dialect for color communication.

This document explains how OptiCAL was changed to comply with the ICC. If you are interested in the technical details, read on. To see some rendered color comparisons and read about our compliance test, skip to Sections V and VI. Otherwise, just be aware that ***the new default is "ICC compatible," but you can still create profiles with the legacy-style colorant tags.***¹

II. Generic ICC Requirement: Transform Screen-White Chromaticity to D50

The easiest Profile Connection Space would have adopted, verbatim, the CIE (X,Y,Z) specification for each commanded color in the source device. If those (X,Y,Z) values were replicated by the destination device, the result would be *exact colorimetric* color reproduction. However, reproducing source-device (X,Y,Z) values does not necessarily reproduce the color's *appearance* on the destination device, because a color's appearance depends on all the other colors seen at the same time. As an estimate of the viewing environment on the source device, the ICC chose the source device's screen white.

The ICC asked the following question. Given a color $\mathbf{X} = (X, Y, Z)^T$ on a screen 1 with white values $\mathbf{X}_W = (X_W, Y_W, Z_W)^T$, and given a screen 2 with white values $\mathbf{X}'_W = (X'_W, Y'_W, Z'_W)^T$, what color $\mathbf{X}' = (X', Y', Z')^T$ on screen 2 will have the same appearance as \mathbf{X} has on screen 1? The ICC adopted an answer based on a chromatic-adaptation model that has been used for a century, originally proposed by Johannes Von Kries (1904). Von Kries's rule has a simple form once one has used a particular matrix \mathbf{M} to transform all the above vectors \mathbf{X} to vectors \mathbf{P} : i.e., $\mathbf{P} = \mathbf{M} \mathbf{X}$, $\mathbf{P}_W = \mathbf{M} \mathbf{X}_W$, $\mathbf{P}' = \mathbf{M} \mathbf{X}'$, and $\mathbf{P}'_W = \mathbf{M} \mathbf{X}'_W$. In that linear basis, the screen-2 tristimulus values P'_k are given by dividing the screen-1 values P_k separately by the values for screen-1 white (P_{Wk}) and then multiplying them by the values for screen-2 white (P'_{Wk}). In other words, von Kries proposed that a human observer sees all tristimulus values in basis \mathbf{P} only in proportion to the white values:

$$P'_k = (P'_{Wk} / P_{Wk}) P_k \tag{1}$$

Eq. (1) represents a diagonal linear transformation on vector \mathbf{X} in the basis \mathbf{P} determined by matrix \mathbf{M} . As shown in Appendix A and also in Wyszecki and

¹ To create profiles with the legacy style colorant tags, choose *Preferences* from the *Edit* Menu, click *Advanced Preferences*, click *ICC Settings*, and set *Chromatic Adaptation Mode* to *None*. Note that, unlike OptiCAL, PhotoCAL will implement only the Bradford-transform default, and will not offer the legacy option.

Stiles (1982, Section 5.12), the Von Kries transformation can be written formally as follows:

$$\mathbf{X}' = \mathbf{M}^{-1} \mathbf{D} \mathbf{M} \mathbf{X}. \quad (2)$$

Here, \mathbf{M} is the 3 x 3 matrix that transforms any (X,Y,Z) values \mathbf{X} to a representation $\mathbf{P} = \mathbf{M} \mathbf{X}$ in which agreed-upon primaries become the unit vectors (1,0,0), (0,1,0), and (0,0,1). The diagonal matrix \mathbf{D} is given by

$$\mathbf{D} = \text{diag}[(\mathbf{M} \mathbf{X}_w')_1 / (\mathbf{M} \mathbf{X}_w)_1, (\mathbf{M} \mathbf{X}_w')_2 / (\mathbf{M} \mathbf{X}_w)_2, (\mathbf{M} \mathbf{X}_w')_3 / (\mathbf{M} \mathbf{X}_w)_3]. \quad (3)$$

For any invertible 3x3 matrix \mathbf{M} , the mapping implied by Eqs. 2 and 3 sends the old screen white to the new screen white:

$$\mathbf{X}_w' = \mathbf{M}^{-1} \mathbf{D} \mathbf{M} \mathbf{X}_w. \quad (4)$$

The ICC chose as its Profile Connection Space (PCS) the space of (X,Y,Z) values transformed as in Eq. (4) from the source-device white \mathbf{X}_w to a standard white \mathbf{X}_w' that corresponds to D50 (a daylight illuminant, at $x = .3457$, $y = .3587$, that has a correlated-color temperature 5000 K). Hence, the ICC convention was to pass (X,Y,Z) values that are “pre-adapted” to a screen white of D50. From the PCS, the destination device could then “adapt” the PCS XYZs to the white of any destination device, again using Eqs. 2 and 3.

Consistent color-management requires that one use the same \mathbf{M} to transform from a monitor to the PCS and back again. If one \mathbf{M} is used to adapt the source XYZs to the PCS and another \mathbf{M} is used to adapt the PCS back to source XYZs, the result will be different colors. Of course, since any \mathbf{M} maps white to the same values, near-neutral colors map nearly correctly, and high-saturation colors show the largest discrepancy.

III. OptiCAL Legacy Colorant-Tag Adaptation

The legacy and new versions of OptiCAL differ in the choice of primaries used to define \mathbf{M} . OptiCAL 3.7 uses an \mathbf{M} that “Von Kries adapts” the tristimulus values in the basis of the device primaries, not in a device-independent basis of human chromatic adaptation. In the legacy software, (X,Y,Z) are adapted to the D50 white by scaling the R, G, and B values of the source monitor so the white has a D50 chromaticity. That amounts to choosing as \mathbf{M} the matrix that brings (X,Y,Z) space into coordinates in which the device primaries lie along the directions (1,0,0), (0,1,0), and (0,0,1). The legacy method is the “best practice” recommended by the National Television Standards Committee for the past 50 years² (Bingley, 1957; Brill, 1992).

² The problem addressed by the N.T.S.C. was to transform all the colors in an image so a white card in a 3200K-lit studio would appear at the 6800K monitor white point (characteristic of CIE illuminant C).

The legacy version of OptiCAL performs the following steps:

- a. Find the (x,y) chromaticities of the RGB primaries and the chromaticity and peak luminance at full activation (screen white). For each primary, scale its Y value so the total Y of screen white is 1. That yields an \mathbf{X} vector for each primary.
- b. From step (a), compute a matrix \mathbf{M}^{-1} that transforms RGB to XYZ. (See Appendix B for a method to perform this computation.)
- c. Compute scale factors S_R, S_G, S_B to multiply the R, G, B values of the primaries, such that the white (x,y) chromaticity at full RGB activation is that of illuminant D50, and so the Y value of that screen white is 1. These factors are the diagonal elements of \mathbf{D} in Eq. (3).
- d. Apply the scale factors in step (c) to scale the measured $(R, G, B) = \mathbf{P}$ of each primary. Transform the result, $\mathbf{D P}$, back to (X, Y, Z) , and thereby obtain $\mathbf{X}' = \mathbf{M}^{-1} \mathbf{D P} = \mathbf{M}^{-1} \mathbf{D M X}$, which is the right-hand side of Eq. (2). Also compute the matrix $\mathbf{M}^{-1} \mathbf{D M}$.
- e. Transmit the transformed device profile, which includes the new screen-white luminance (at the D50 chromaticity) and the chromaticities of the primaries. In subsequent color management, the digital values from the source device are treated as if they came from a device with the transformed white point and primaries. That process is equivalent to multiplying all measured (X, Y, Z) vectors by $\mathbf{M}^{-1} \mathbf{D M}$ before transmitting them as pixel values. It is also equivalent to multiplying the (R, G, B) vectors by \mathbf{D} and then transforming the scaled values to (X, Y, Z) values.

In the above algorithm, the von Kries transformation scales the device R, G, B primaries separately. The gain change of one primary alters the intensity but not the chromaticity of the primary. Hence the (x,y) chromaticities of the primaries are sent without modification to other applications.

[Note: Although neither perceptually based nor ICC-standard, the legacy OptiCAL's colorant-tag transformation has one advantage: When applied to additive three-primary devices, it transforms the entire source gamut (RGB triangle) to the entire destination gamut. The destination device simply drives that device's primaries as if they were the primaries of the source device, but with gains chosen to achieve the desired white point.]

IV. New OptiCAL Colorant-Tag Adaptation with Bradford Transform

The ICC prescribes a matrix \mathbf{M} that transforms each CIE tristimulus vector \mathbf{X} to a basis \mathbf{P} that is device-independent and represents the scaling of human visual responses in chromatic adaptation. Three choices of "vision-based" transform are given to an ICC user: XYZ, Bradford RGB, and a basis that is called simply "Von Kries." The XYZ option simply adopts the identity matrix for \mathbf{M} , and scales the X, Y, and Z coordinates to achieve the D50 white. That option is used in defining CIELAB color space, but is now acknowledged to be "wrong Von Kries"

(Fairchild, 1997; ICC, 2001). ICC also does not recommend the generic Von Kries basis, which is a holdover from comparative research studies by Hunt, Pointer, and Estevez (see Fairchild, 1997). That leaves the Bradford transform, which is favored as the proper one for ICC use. Any Bradford transform (of the form $\mathbf{M}^{-1} \mathbf{D} \mathbf{M}$) is a diagonal linear transformation along three special axes that have chromaticities $(x_r, y_r) = (0.6996, 0.3064)$, $(x_g, y_g) = (-0.3575, 1.261)$, $(x_b, y_b) = (0.1358, 0.0419)$. The Bradford \mathbf{M} matrix, derived from extensive appearance-matching experiments (for background, see Süssstrunk et al., 2001), is given by

$$\mathbf{M}_{\text{BFD}} = \begin{array}{c} | 0.8951 \quad 0.2664 \quad -0.1614 | \\ | -0.7502 \quad 1.7135 \quad 0.0367 | \\ | 0.0389 \quad -0.0685 \quad 1.0296 | \end{array} .$$

An ICC-compliant color-management system will present its colors on a monitor with a given white point (call it W , which could be D65) by Bradford-transforming the recorded \mathbf{X} values from D50 to W . Call this transform $\mathbf{BFD}(W|50) = \mathbf{M}_{\text{BFD}}^{-1} \mathbf{D}(W|50) \mathbf{M}_{\text{BFD}}$, which involves the Bradford matrix \mathbf{M}_{BFD} and the diagonal illuminant-transform matrix $\mathbf{D}(W|50)$.

To “precorrect” for subsequent color management, the new OptiCAL (version 3.7.6) transforms raw measured (X,Y,Z) vectors to the D50 space using the inverse of $\mathbf{BFD}(W|50)$, which is $\mathbf{BFD}(50|W) = \mathbf{M}_{\text{BFD}}^{-1} \mathbf{D}(50|W) \mathbf{M}_{\text{BFD}}$. The steps of the process are as follows:

- a. Find the (x,y) chromaticities of the RGB primaries and the chromaticity and peak luminance of screen white. Scale all the primaries together so the Y of screen white is 1. That yields an \mathbf{X} vector for each primary.
- b. Retrieve stored values of the Bradford matrix \mathbf{M}_{BFD} . Then compute $\mathbf{P} = \mathbf{M}_{\text{BFD}} \mathbf{X}$ for each primary, and also for the screen white.
- c. Compute scale factors S_R, S_G, S_B that will multiply the \mathbf{P} values of the primaries, such that the (x,y) chromaticity of white at full RGB activation is that of illuminant D50, and so the Y value of that screen white is 1. These factors are the diagonal elements of \mathbf{D} in Eq. (3).
- d. Apply the scale factors in step (c) to scale the vector \mathbf{P} of each primary, found in step b. Transform the result, $\mathbf{D} \mathbf{P}$, back to (X,Y,Z) , and thereby obtain $\mathbf{X}' = \mathbf{M}_{\text{BFD}}^{-1} \mathbf{D} \mathbf{P} = \mathbf{M}_{\text{BFD}}^{-1} \mathbf{D} \mathbf{M}_{\text{BFD}} \mathbf{X}$, which is the right-hand side of Eq. (2). Also compute the matrix $\mathbf{M}_{\text{BFD}}^{-1} \mathbf{D} \mathbf{M}_{\text{BFD}}$.
- e. Transmit the transformed device profile, which includes the new peak luminance (relative to D50) and the chromaticities of the transformed primaries. In subsequent color management, the digital values from the source device are treated as if they came from a device with the transformed white point and primaries. That process is equivalent to multiplying all measured $(X,Y,Z)^T$ vectors by $\mathbf{M}_{\text{BFD}}^{-1} \mathbf{D} \mathbf{M}_{\text{BFD}}$ before transmitting them as pixel values.

The above process prepares input (X,Y,Z) values from the source monitor so that an ICC-compliant color-management system will render the same (X,Y,Z) values on an identical destination monitor. Also, the measured (x,y) values of the primaries on the latter will match the profile values from the former.

V. Comparison of Rendered Colors Using Legacy and New OptiCAL.

Here are test patterns showing visual differences between legacy OptiCAL monitor profiles and new OptiCAL monitor profiles using the Bradford transform for chromatic adaptation of the colorant tags. In each case Adobe Photoshop interprets the profile to adapt to a white point that is close to D65.

Figure 1. Natural image viewed with the Legacy profile:



Figure 2. Natural image viewed with the New profile:



Figure 3. Color patches viewed with the Legacy profile:



Figure 4. Color patches viewed with the New profile:



Figure 5. Viewing color patch difference wedges side by side (boundary in each square extends from upper right to lower left):



The above figures---created in Photoshop and imported into Word---suggest that the change from old to new profiles does not much change the appearance of either natural images or patches. The instrument-measured color-difference values for the above patches ranged from negligible (for the grays, patches 19-24) to about 4 units of CIELAB ΔE^* for patches 13 and 14 (saturated blue and green, respectively).

VI. Test of Compliance

ColorVision designed a test of ICC compliance for colorant-tag adaptation, and subjected to the test the legacy version of OptiCAL, the new version of OptiCAL (that uses the Bradford transform), and two other popular monitor calibration software packages offered by other suppliers. The test consisted of the following steps:

1. Obtain an ICC-compliant application (we chose Version 7.0 of Adobe Photoshop).
2. On a single test monitor (the Mitsubishi 2040u), measure the (x,y) values of the white point and the colors of full-screen R, G, B primaries at peak activation. The measurements are performed using the colorimeter that is sold with the software. (For example, the Spyder is used with OptiCAL.) The measured screen-white chromaticity was close to that of D65.

3. Use the calibration software to create a profile for the test monitor. Following the ICC rules, record the native white chromaticity of the device and the chromatically adapted colorant tags (adapted to D50). Store the profile that was thereby created.
4. Launch the ICC-compliant application (in this case, Photoshop 7.0).
5. Steps 5-7 are particular to Photoshop, but may have counterparts in other ICC-compatible applications. In the “Color Settings” dialog box, select under “RGB Working Spaces” the profile that was just stored for the test monitor in Step 3. Then, again under “RGB Working Spaces”, select “Custom RGB.”
6. The default profile parameters that appear in the “Custom RGB” dialog are derived from the values in the stored profile. Photoshop records the native white point without change, but the colorant tags are chromatically adapted from a D50 white point to the native white point. Photoshop uses a Bradford transform, as recommended by the ICC.
7. Compare the colorant tags (R, G, B chromaticities) in the “Custom RGB” dialog with those originally measured on the monitor. If the profiling software is ICC compliant, the colorant-tag chromaticities should be identical.

[NOTE: Because Photoshop makes available a dialog box that shows the native white point and colorant tags they have “adapted back” to the native state of the monitor, a user can see whether the profiling software under test is providing a transform that is “understood” and inverted by Photoshop. For ICC-compatible software that lacks such a utility, a second screen measurement would be necessary to determine if the commanded RGB chromaticities are the same as the native ones.]

The following results were obtained from the compliance test:

1. The legacy version of OptiCAL (Version 3.7) did not pass this test, showing Δx and Δy magnitudes as large as 0.016.
2. The two other software packages also failed the compliance test, showing Δx and Δy magnitudes as large as 0.023.
3. The new version of OptiCAL (Version 3.7.6) passed the test with Δx and Δy magnitudes less than 0.0005---i.e., below round-off error in the numerical presentation.

The results appear in detail in the table below:

Photoshop Color Settings Test of Monitor Profile

OptiCAL 3.7.6 (New)

	R		G		B		W	
	x	y	x	y	x	y	x	y
Measured	0.626	0.352	0.277	0.600	0.138	0.069	0.314	0.323
Photoshop	0.626	0.352	0.277	0.600	0.138	0.069		
Difference	0.000	0.000	0.000	0.000	0.000	0.000		

OptiCAL 3.7 (Legacy)

	R		G		B		W	
	x	y	x	y	x	y	x	y
Measured	0.632	0.353	0.277	0.604	0.138	0.066	0.314	0.323
Photoshop	0.625	0.353	0.261	0.590	0.137	0.060		
Difference	0.007	0.000	0.016	0.014	0.001	0.006		

Software Product 1

	R		G		B		W	
	x	y	x	y	x	y	x	y
Measured	0.616	0.342	0.282	0.605	0.148	0.075	0.315	0.327
Photoshop	0.614	0.341	0.272	0.620	0.172	0.080		
Difference	0.002	0.001	0.010	-0.015	-0.023	-0.005		

Software Product 2

	R		G		B		W	
	x	y	x	y	x	y	x	y
Measured	0.626	0.339	0.287	0.605	0.147	0.071	0.315	0.323
Photoshop	0.627	0.343	0.280	0.623	0.170	0.076		
Difference	-0.001	-0.004	0.007	-0.018	-0.023	-0.005		

VII. Nuances of *Wtpt* and *Chad* Tags

Some explanation is due about the compatibility of OptiCAL with two versions of ICC specification: 2.0 and 4.0. In Version 2.0, the profile of an additive 3-color display contains the measured white point (the *wtpt* tag) and the chromatically adapted colorant tags (x and y values of the R, G, B primaries). In Version 4.0, a profile contains the 3-by-3 matrix of chromatic adaptation (the *chad* tag) and the unadapted (x,y) values of the primaries, but not the measured white point. The default in both OptiCAL and PhotoCAL is to create ICC profiles based on the ICC 2.0 specification, so the measured white point is stored in the *wtpt* tag.

Photoshop 7.0, used in our compliance test, is compatible with both ICC 2.0 and 4.0, and hence compatible with the default settings of the new version of OptiCAL. If Photoshop sees a *wtpt* tag and no *chad* tag, it calculates the values in the dialog by transforming from D50 to the white point indicated in the *wtpt* tag. If there is a *chad* tag, then Photoshop uses the chromatic-adaptation matrix for the transform and ignores the *wtpt* tag.

The interaction of Photoshop 7.0 with the new OptiCAL default suggests that the ICC-2.0 default (that is, to create ICC 2.0 profiles using the Bradford transform) should suffice for good color communication. However, to accommodate exceptional applications, the new version of OptiCAL includes two sets of options in the "ICC Settings" dialog. One set allows the user to specify profiles that adhere to either the 2.0 or the 4.0 specification (4.0 profiles include the *chad* tag). The other set allows selection of chromatic adaptation for the colorant tags: Legacy, Bradford, or XYZ Scaling. The user is strongly urged to leave the settings at default---that is, to create ICC 2.0 profiles using the Bradford transform.

VIII. Conclusions

Monitor profiles created by the new version (3.76) of OptiCAL contain colorant tags that are scaled to a D50 screen white using the Bradford transform. Legacy versions of OptiCAL scaled the device primaries, not the Bradford primaries, to a D50 white. The differences in rendered color are small, particularly for near-neutrals. However, the adjustment was necessary for consistent communication with ICC-compliant applications. A compliance test showed that the required transform had been implemented correctly. Among screen-calibration software suites, OptiCAL is one of the first to have implemented the correct colorant-tag adaptation relative to the Photoshop application.

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APPENDIX A: Dependence of Von Kries Transformation on Choice of Primaries

In a particular tristimulus coordinate system, the Von Kries transformation is a diagonal linear transformation. In that coordinate system, let $\mathbf{P} = (P_1, P_2, P_3)^T$, $\mathbf{P}_W = (P_{W1}, P_{W2}, P_{W3})^T$, and $\mathbf{P}'_W = (P'_{W1}, P'_{W2}, P'_{W3})^T$ be the tristimulus vectors of a test color, an old screen white, and a new screen white, respectively. (Here, all vectors are column vectors, so the row-vector representation in the text is denoted by the matrix transpose T .) It is desired to find the tristimulus vector of the color $\mathbf{P}' = (P'_1, P'_2, P'_3)^T$ which, in the environment of the new screen white, appears the same as \mathbf{P} appears in the environment of the old screen white. The Von Kries model posits that the appearance depends on the ratio of the test to screen-white tristimulus values, so that equating the appearance means

$$\begin{aligned} P'_1/P'_{W1} &= P_1/P_{W1} \\ P'_2/P'_{W2} &= P_2/P_{W2} \\ P'_3/P'_{W3} &= P_3/P_{W3} . \end{aligned} \tag{A1}$$

Eq. (A1) can be solved for \mathbf{P}' to give

$$\begin{aligned} P'_1 &= [P'_{W1}/P_{W1}] P_1 \\ P'_2 &= [P'_{W2}/P_{W2}] P_2 \\ P'_3 &= [P'_{W3}/P_{W3}] P_3 . \end{aligned} \tag{A2}$$

Eq. (A2) can be expressed in matrix form as

$$\mathbf{P}' = \mathbf{D} \mathbf{P}, \tag{A3a}$$

where

$$\mathbf{D} = \text{diag}[P'_{W1}/P_{W1}, P'_{W2}/P_{W2}, P'_{W3}/P_{W3}]. \tag{A3b}$$

Now, how does Eq. (A3) show up in the CIE XYZ system? Suppose, for each vector \mathbf{P} , the corresponding \mathbf{X} is given by $\mathbf{X} = \mathbf{M}^{-1} \mathbf{P}$. Then Eq. (A3a) can be rewritten as follows in terms of \mathbf{X} quantities:

$$\mathbf{M} \mathbf{X}' = \mathbf{D} \mathbf{M} \mathbf{X}. \tag{A4}$$

Premultiplying Eq. (A4) by \mathbf{M}^{-1} yields Eq. (2) in the main text:

$$\mathbf{X}' = \mathbf{M}^{-1} \mathbf{D} \mathbf{M} \mathbf{X}, \tag{A5}$$

Similarly, matrix \mathbf{D} from Eq. (A3b) can be rewritten in terms of quantities \mathbf{X} to give

$$\mathbf{D} = \text{diag}[(\mathbf{M} \mathbf{X}_w')_1/(\mathbf{M}\mathbf{X}_w)_1, [(\mathbf{M} \mathbf{X}_w')_2/(\mathbf{M}\mathbf{X}_w)_2, [(\mathbf{M} \mathbf{X}_w')_2/(\mathbf{M}\mathbf{X}_w)_2]. \quad (\text{A6})$$

For further information, see www.brucelindbloom.com, under “Math” and “Chromatic Adaptation.”

APPENDIX B. Transformation of (RGB) to (XYZ).

This Appendix describes how to compute CIE 1931 (X, Y, Z) values from monitor (R,G,B) values, given the chromaticity coordinates (x_r, y_r), (x_g, y_g), (x_b, y_b) of the three device primaries, and the chromaticity of the monitor white point (x_w, y_w). The following equations apply:

$$\begin{aligned} X &= R x_r Y_{or}/y_r + G x_g Y_{og}/y_g + B x_b Y_{ob}/y_b \\ Y &= R Y_{or} + G Y_{og} + B Y_{ob} \\ Z &= R z_r Y_{or}/y_r + G z_g Y_{og}/y_g + B z_b Y_{ob}/y_b. \end{aligned} \quad (\text{B1})$$

Here, z_r = (1-x_r-y_r), z_g = (1-x_g-y_g), z_b = (1-x_b-y_b), and Y_{or}, Y_{og}, Y_{ob} are given by

$$\begin{vmatrix} Y_{or} \\ Y_{og} \\ Y_{ob} \end{vmatrix} = \begin{vmatrix} x_r/y_r & x_g/y_g & x_b/y_b \\ 1 & 1 & 1 \\ z_r/y_r & z_g/y_g & z_b/y_b \end{vmatrix}^{-1} \begin{vmatrix} x_w/y_w \\ 1 \\ z_w/y_w \end{vmatrix}, \quad (\text{B2})$$

where z_w = (1-x_w-y_w). (See Post, 1992.)

The tristimulus values X_w, Y_w, Z_w of the device's white point, correspond to the chromaticity (x_w, y_w) and are such that, at full activation, Y_w = 1; these values are (X_w, Y_w, Z_w) = (x_w/y_w, 1, z_w/y_w).

Note that Eq. (B1), with the substitution of (B2), provides an expression for the legacy-OptiCAL matrix \mathbf{M}^{-1} in the main text.